Flow Distribution in Manifolds for Low Reynolds Number Flow

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(Received February 10, 1997)

This paper addresses the fundamental flow distribution question of how to design manifolds of low Reynolds number flow with both numerical analysis and experiments. The present study introduces new parameters of α_c and α_d, defined as the ratio of header diameter to header length in combining and dividing manifolds, the parameters which are not clearly considered in the previous studies of flow distribution in manifolds. The parameters of α_c and α_d were found to govern the flow distribution independently of each other. Varying α_c, α_d, and the Reynolds number respectively, a correlation of optimal flow distribution is obtained for laminar fow in manifolds as follows; $\alpha_d \cdot Re_w^m = K$ where ac $\alpha_c \geq 1/4$. The proposed correlation makes predictions possible for wide ranges of α_d and Re_w. Also, the present numerical results show satisfactory agreements with those of flow visualization. From the flow visualization, recirculating flow regime was observed at the inlet of each channel, in which hot spots may occur due to small velocities. The size of recirculating flow regime is strongly dependent on the Reynolds number and is smaller for optimal cases than others.

Key Words: Manifolds, Optimal Flow Distribution, Area Ratio, Width Ratio, Ratio of Diameter to Length in the Headers

Nomenclature

AR : Area ratio ($n \cdot D_{ch}/D_d$ or L/D_d)
D_c : Diameter of combining header
D_d : Diameter of dividing header
H : Channel length
K : Constant in Eq. (6)
L : Header length
m : Exponent constant in Eq. (6)
Q_i : Flow rate of a channel
Re_D : Reynolds number based on the dividing header diameter, $\frac{\rho V_{in} D_d}{\mu}$
Re_w : Reynolds number based on a channel width, $\frac{\rho V_{in} w}{\mu}$
V_{in} : Inlet velocity
W : Width of a channel
WR : Width ratio

Greek symbols

α_c : Ratio of diameter to length in combining header ($=D_c/L$)
α_d : Ratio of diameter to length in dividing header ($=D_d/L$)
δ : Wall thickness
ν : Kinematic viscosity
μ : Dynamic viscosity

Subscripts

t : Total

1. Introduction

In this paper we consider a fundamental problem of the uniform flow distribution, which has important applications in the area of electronic cooling module design. The problem consists of eliminating a local hot spot by providing uniform

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flow distribution on each channel. To obtain the optimal distribution, proper consideration must be given to flow behavior according to the geometric shape factor of headers. It is well known that the flow behavior is governed by force balance between the flow inertia and the friction in flow branching. In a dividing header, the main fluid stream is decelerated due to the loss of both fluid and momentum through channels. This causes a rise in pressure in the direction of flow if we apply a frictionless Bernoulli equation to the header flow. However, the frictional effect would cause a decrease of pressure in the direction of flow.

For a higher Reynolds number flow such as turbulent flow, the characteristics of flow distribution are comprehensively investigated by Bajura and Jones (1976). They examined each effect of various parameters such as area ratio, flow resistance of the channel, length/diameter ratio of header, momentum parameters, width ratio, diameter ratio of header to channel, and friction factor on the flow distribution with analytical solutions and experiments. Their results provided a wide range of design rule for manifolds with the high Reynolds number flows. There were other published works (Riggs, 1987: Datta, 1980: Shen, 1992: Kubo, 1969) which made advances in the design of manifolds.

For high Reynolds number flow conditions in parallel flow manifolds, flow distribution curves tend to display monotonical increase in the direction of the flow. However, the general trends observed at high Reynolds number flows are not applicable for most lower Reynolds number flow conditions, which show downward convex parabolic distribution curves and show the minimum flow rate in the middle of channels. The low Reynolds number flow conditions commonly occur in small heat exchangers or electronic cooling modules. Therefore, low Reynolds number flows would require a new set of studies to obtain the design rule for optimum manifold flow distribution.

Choi et al. (1993a) studied the laminar flow distribution in manifolds of liquid cooling module for electronic packaging. They reported that the distribution curves were strongly affected by the area ratio: decreasing the diameter of the dividing header caused the change of distribution curve from monotonical increase to parabolic profile. It is theoretically true that a flow distribution curve can be changed to almost horizontal by increasing the width ratio, which is the ratio between headers reported by Choi et al. (1993b). Increasing the width ratio more than 2, however, is not practical nor desirable bacause there may be larger recirculting flow regions in the corner of manifold. Bassiouny and Martin (1984a, 1984b) conducted an analytical study of the flow distribution in the plate heat exchangers and obtained a general parameter(m) which determines the flow distribution: when m^2 approaches zero, the flow distribution becomes uniform.

Most of the previous optimization works (Riggs, 1987: Datta, 1980: Shen, 1992: Kubo, 1969) were about pipe manifolds with discontinuous branches and high Reynolds numbers so that the distribution characteristics were quite different from those for laminar flow. Therefore, the optimization of laminar flow distribution needs to be done. Focusing now on the task of optimizing and delineating the laminar flow distribution in the manifolds of Fig. 1 using the main parameters including area ratio(L/D) and width ratio (a_c/a_d). The objective of the present study is to conduct the optimization work which mainly consists of the following questions;

(i) Is there any limit of explanation for flow distribution using the above-mentioned parameters ?

(ii) Is there any other parameter which governs the flow distribution ?

(iii) Is it possible to correlate all the optimization cases into a simple (compact) formula that can have wide applicability ?

2. Mathematical Formulation and Numerical Method

To optimize the manifold geometry in an intermediate range of laminar flow conditions, we developed an efficient way of examining the effect of the manifold's geometry on the flow distri-

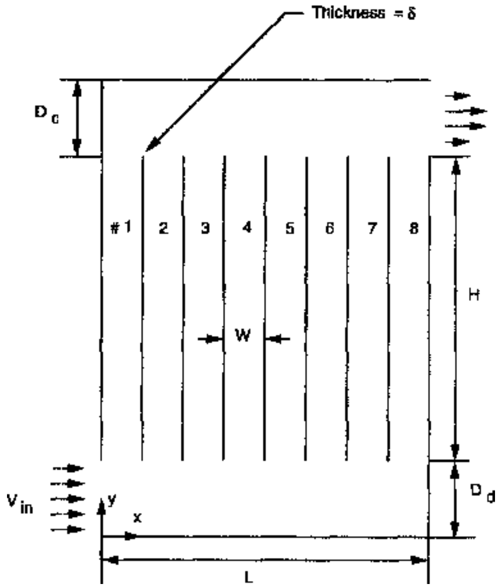


Fig. 1 Top view of the manifold.

bution. Hence, we studied a wide range of geometric modifications until we arrived at the designs in which the respective flow distribution were nearly the same. We accomplished this process by numerical simulation of the flow fields in the two dimensional domain as shown in Fig. 1. For the numerical work, we chose multi-channels with a square cross section, i. e., L = H, because the effect of H/L on the optimized geometry was found negligible. The plate thickness was fixed as $\delta/W = 1/20$.

The flow in the manifolds was considered as laminar, and water was selected for the working fluid with its constant properties referred at 20℃. The governing equations used for the present numerical study were as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial P}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \tag{2}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial P}{\partial y}+\nu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) \tag{3}$$

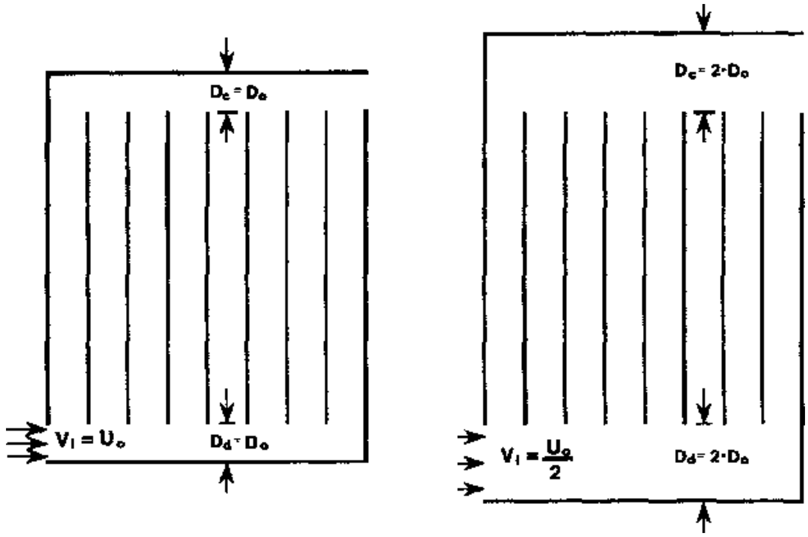
The calculations were performed using a finite volume software packages, Fluent (Version 4. 3). The package had demonstrated benchmark problems which were the same type of flow as the present one. Those results can be found elsewhere (Choi, 1993a, 1993b: Kim, 1995). For manifolds with eight channels, grid independent solutions were obtained with 252×274 grids. Non-uniform grids were used for both x and y directions, with fine grids near the walls and branch points. The sum of normalized residuals which offer a measure of the magnitude of error in the solution at each iteration was set to be 10^{-3} as convergence criteria. The standard SIMPLE algorithm was used in this calculation.

3. Limits of Previous Parameters

The first design question we addressed was how appropriate the previous governing parameters were in delineating the flow distribution mechanism. Previous parametric studies (Choi, 1993a, 1993b) using AR, WR, and the Reynolds number provided important information for design guides. However, their effects were somewhat combined with each other so that it became difficult to interpret the results. Figure 2 shows a comparison of the inlet condition and geometry with varying area ratio for the constant WR and Re_D in the previous study (Choi, 1993b). It is of note that increasing the dividing header diameter (D_d) decreases the area ratio(L/D_d) and increases the Reynolds number (Re_D). In order to keep the Reynolds number constant, it is necessary to decrease the inlet velocity, which result in different flow conditions.

From these configurations we expect that the flow rate in Fig. 2 remains constant rather than the Reynolds number. In fact, the Reynolds number (Re_W) based on the channel-width decreases from 100 to 50. For a fixed width ratio, symmetric changes in both headers were assumed. Due to the symmetric change of headers, the effect of a single header on flow distribution was overlooked. All the problems described above exist in the use of variable dividing header diameter as the reference length for Reynolds number (Re_D), area ratio, and width ratio. Therefore, it is necessary to find other governing parameters whose reference length are not variable.

For this reason, we present new but well known parameters in manifolds geometry. An



(a) AR=8 (WR=1, Re_D=100, Re_W=100) (b) AR=4 (WR=1, Re_D=100, Re_W=50)

Fig. 2 Comparison of the inlet condition and geometry for varying AR for the same WR and Re_D.

aspect ratio for combining headers, α_c, is defined as the ratio of header diameter to header length. Similarly, an aspect ratio for dividing headers, α_d, is defined as the ratio of the dividing header diameter to header length. In fact, α_d is the inverse of the previous parameter of AR, whereas α_c is a ratio of WR to AR as shown in the following equations :

$$\alpha_d = \frac{1}{AR} = \frac{D_d}{L} \quad (4)$$

$$\alpha_c = \frac{WR}{AR} = \frac{D_c/D_d}{L/D_d} = \frac{D_c}{L} \quad (5)$$

4. Effects of Parameters on Flow Distribution

After we fix the new parameters required to conduct an optimization, we investigated the second design aspect, which was to see the effects of the governing paramters on flow distribution and how appropriate they were for delineating the combined effects.

Figure 3 shows the effect of α_d on the flow distribution curves at Re_W=100. For a reltivlely large α_d (=1/4), the flow distribution is biased to the last channel. However, that for a reltivlely small α_d (=1/16) is biased to the first channel. As we decrease α_d, the flow rate near the last channel is significantly reduced and the flow rate near the first channel is increased. Also, the flow distribution profile becomes parabolic which does not occur in the high Reynolds number flow conditions. It is of note that if there exists a parabolic curve of flow distribution, it is difficult to obtain uniform distribution by adjusting the Reynolds number only.

The phenomenon in Fig. 3 can be interpreted as follows. Friction strongly depends on α_d and causes the pressure drop. Meanwhile, fluid loss through a dividing header causes momentum loss, which results in pressure recovery along the header. Also, the flow inertia related with the Reynolds number influences the flow behavior in both dividing and combining headers. As α_d is decreased, friction becomes dominant, and the pressure decreases in the direction of flow in low Reynolds number ranges. Meanwhile, for a high Reynolds number flow, the effect of friction on pressure becomes relatively small compared with those of fluid loss and flow inertia, so that the pressure profile in a dividing header increases in the direction of flow (Bajura and Jones, 1976).

One interesting aspect of flow distribution depicted in Fig. 3 is that the present results are

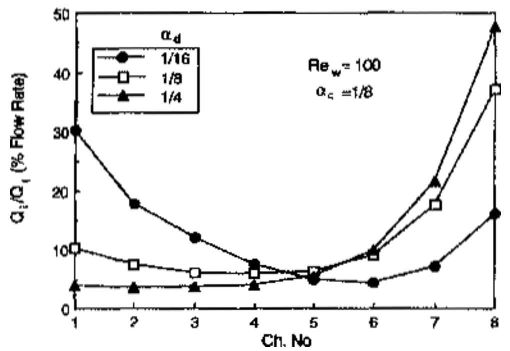


Fig. 3 Flow distribution for three different ad for $Re_w = 100$ and $\alpha_c = 1/8$.

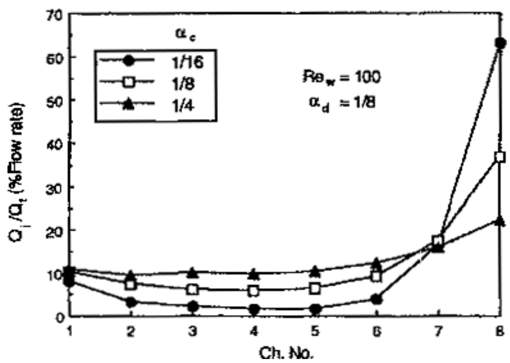


Fig. 4 Flow distribution for three different α_c for $Re_w = 100$ and $\alpha_d = 1/8$.

very similar to the previous results (Choi, 1993a) which investigated the effect of area ratio on the flow distribution. The present study varies the dividing header diameter for a constant α_c, whereas Choi et al. (Choi, 1993a) varied AR for a fixed WR. In fact, there is a slight difference in the Reynolds numbers between two studies due to different characteristic lengths. If we compensate the Reynolds number difference, the flow distribution curves of two results almost coincident each other. The comparison provides an implicit but important idea that the area ratio can be replaced with α_d. As shown in Eq. (4), α_d is the inverse of the area ratio, which has the same physical meaning.

As described earlier in Fig. 2, AR in the previous analysis (Choi, 1993a) included several combined effects of α_d, α_c and Re. On the other hand, if the previous analysis had used the conditions givien in the present study as in Fig. 3, there would have been a combined effect of area ratio and width ratio due to varying D_d and asymmetric headers. Therefore, the good agreement between the previous study (Choi, 1993a) and the present results in Fig. 3 implies that the common factor in both analyses is the main governing parameter of flow distribution, α_d. For the case given in Fig. 3, the area ratio or α_c is not so much dominant in flow distribution. However, this does not mean that the area ratio or α_c is not the governing paramter of flow distribution in manifolds.

Figure 4 shows the effect of α_c on the flow distribution. Equation (5) provides the physical meaning of α_c related with the width ratio and area ratio. For a fixed area ratio and the Reynolds number, α_c has the exact same meaning of width ratio, so that the results in Fig. 4 are exactly the same as Choi et al. results (Choi, 1993b). In Fig. 4, as we increase α_c, the flow rate of the last channel decreases significantly, and the flow rate is evenly distributed to all channels. However, the flow rate in the first channel almost does not change. This suggests that the combination of the effects of α_c in Fig. 4 and α_d in Fig. 3 may adjust the flow distribution for optimum. But, after examining various α_c, we found that α_c behaved not as a control parameter but a criterion to get the optimal flow distribution, a phenomenon which will be discussed in the next section.

Meanwhile, we investigated the effect of channel length (H) on flow distribution. As expected, the channel length did not play an important role in flow distribution because the flow resistance due to the channel length was relatively small compared with that caused by the header. If flow resistances in channels become large, for example, using orifices, uniform distribution will be easily obtained. Large flow resistance in a channel, however, results in a high total pressure drop for the manifold system, a trend which may not be acceptable if pumping cost is an important design consideration. Furthermore, we also investigated the effect of the number of channels on flow distribution. The result showed no significant change in the flow distribution profile.

5. Optimal Conditions of Flow Distribution

We investigated whether the flow parameters in the manifolds can be selected optimally so that the variation in flow rate is minimized. For an aspect ratio of the dividing header (α_d), we found a monotonically increasing curve of flow distribution, which might eventually be uniform by varying α_c and Reynolds numbers. Through examining various α_c, it is found that α_c strongly influences the flow distribution. Beyond a certain value of α_c, it did not seem to be dominating the flow distribution, indicating that α_c played not as a control parameter but a criterion to get the optimal flow distribution. Therefore, we focused on finding the optimal α_d for a given flow rate or the Reynolds number. However, since varing the Reynolds number for a fixed α_d was much easier for conducting numerical analyses, we tried to find the optimal Reynolds number for a given α_d. In other words, with varying Reynolds numbers, we examined the flow distribution curves.

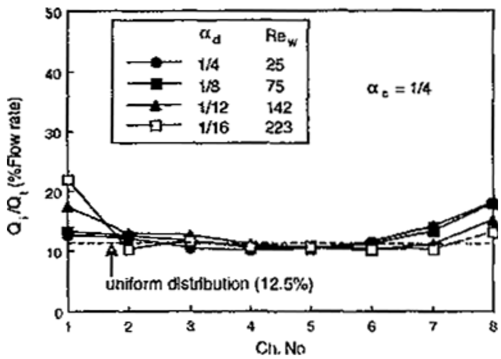


Fig. 5 Optimized flow distribution for various α_d and Re_w.

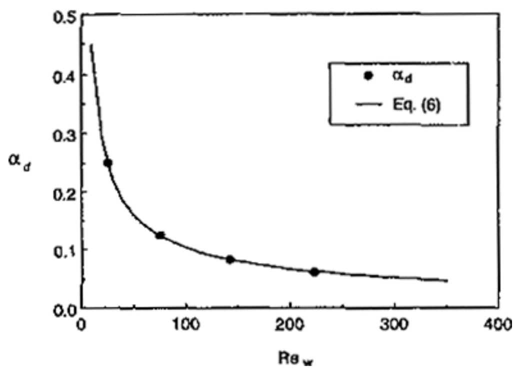


Fig. 6 Correlation of flow parameters for optimization.

It was clear that there existed an optimal Reynolds number for flow distribution. Still, the resulting curves were not so flat, but flow rate variations were significantly minimized for the optimal Reynolds number. Therefore, we determined the optimal Reynolds number for each α_d. The results are presented in Fig. 5, for the ranges of $1/16 \leq \alpha_d \leq 1/4$ and $25 \leq Re_w \leq 250$.

In order to correlate $(Re_w)_{\rm opt}$ or $(\alpha_d)_{\rm opt}$ results, we plotted the results in the form of α_d vs. Re_W as shown in Fig. 6. The result produced the following optimal correlation between the aspect ratio of dividing header and Reynolds number for manifolds :

$$\alpha_d \, Re_w^m = K \, (\alpha_c \geq 1/4) \tag{6}$$

where $m=0.64$ and $K=1.97$. Equation (6) shows that the Reynolds number is inversely proportional to α_d. In other words, for small α_d, the Reynolds number should be large, and vice versa.

Meanwhile, α_c is not directly involved in Eq. (6). In fact, increasing α_c results in a favorable flow distribution as well as a reduction of total pressure loss in the manifold system. However, it results in a large recirculating area in corners and requires large outlet diameter, which is not practical. In addition, increasing α_c above a certain value does not affect much the flow distribution. Therefore, after examining various α_c for optimized conditions in Eq. (6), we set the generally acceptable criteria for α_c which is $\alpha_c \geq 1/4$. The problem of recirculation for large α_c can be solved by using the headers of trapezoidal shape (Kim, 1995).

6. Comparison with Flow Visualization

We conducted flow visualization experiments with equidistantly positioned 8 channels of parallel manifolds (*i. e.*, $L \times H \times b = 280\text{mm} \times 310\text{mm} \times 30\text{mm}$). The test section was made of plexiglass to take a picture of flow distribution. The hydraulic diameters of dividing and combining headers were 20 mm and 40 mm, respectively. Based on geometry, the governing parameters of flow distribution are as follows: $\alpha_d = 1/12$ and $\alpha_c = 1/6$, resulting in AR=12 and WR=2, respectively. The schematic diagram of experimental apparatus is shown in Fig. 7. A uniform inlet velocity profile was applied in the experiments, which was the same condition in the numerical analysis. In order to obtain the uniform inlet velocity profile, a sudden contraction was used right after a calming chamber. In order to minimize the fluctuating components of flow, we used a constant overhead water tank instead of a pump. A dye-tank was located slightly higher than the overhead tank and the dye was injected through the needles into the channel flow. Due to the injected dye, the flow system was designed as an once-through system. For each experiment, the flow rate was measured with a balance.

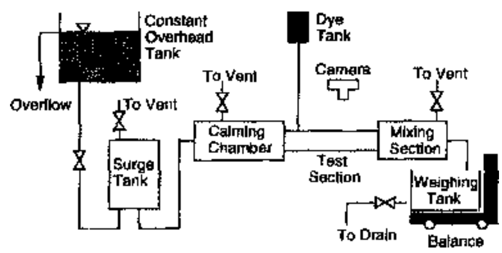


Fig. 7 Schematic diagram of experimetal apparatus for flow visualization.

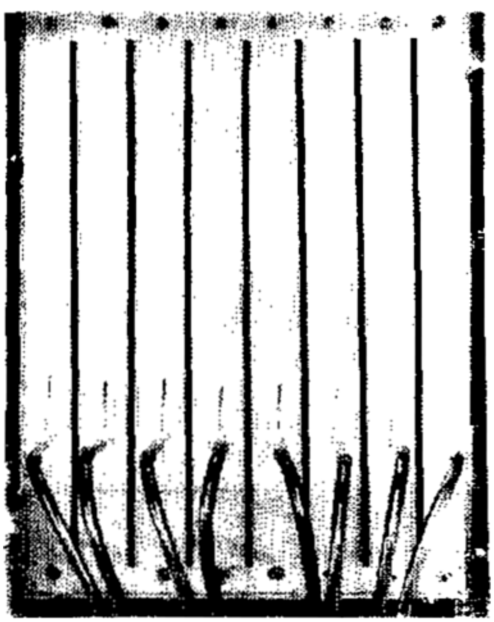


Fig. 8 Flow visulaization with dye injection for $Re_w = 300$.

Figure 8 shows a typical flow distribution for $Re_w = 300$ obtained from the dye-injected flow visualization. The last channel shows the fastest velocity among 8 channels whereas the other channels show approximately the same level of velocities. We found that this trend was increasing with increasing the Reynolds number. The results of flow visualization are compared with the present numerical results in Fig. 9, which shows an excellent agreement between the dimensionless center velocities obtained from the experiment and the computation for $Re_w = 450$ at each channel. In the experiment, the center velocities in the channels were obtained by measuring the lengths of streamlines per unit time on the pictures and were non-dimensionalized for the comparison with experiments. Both of numerical and experimental results show a typical parabolic distribution profile, a phenomenon which is unique for low Reynolds number flow.

One interesting observation in the present analysis was that there was a significant area of recirculation on each channel. Commonly, a recirculating flow zone had low velocities when compared with the main flow, causing hot spots

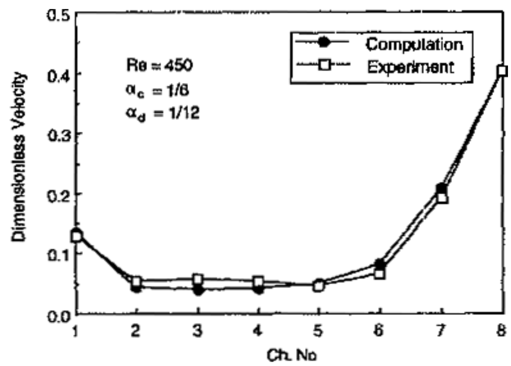
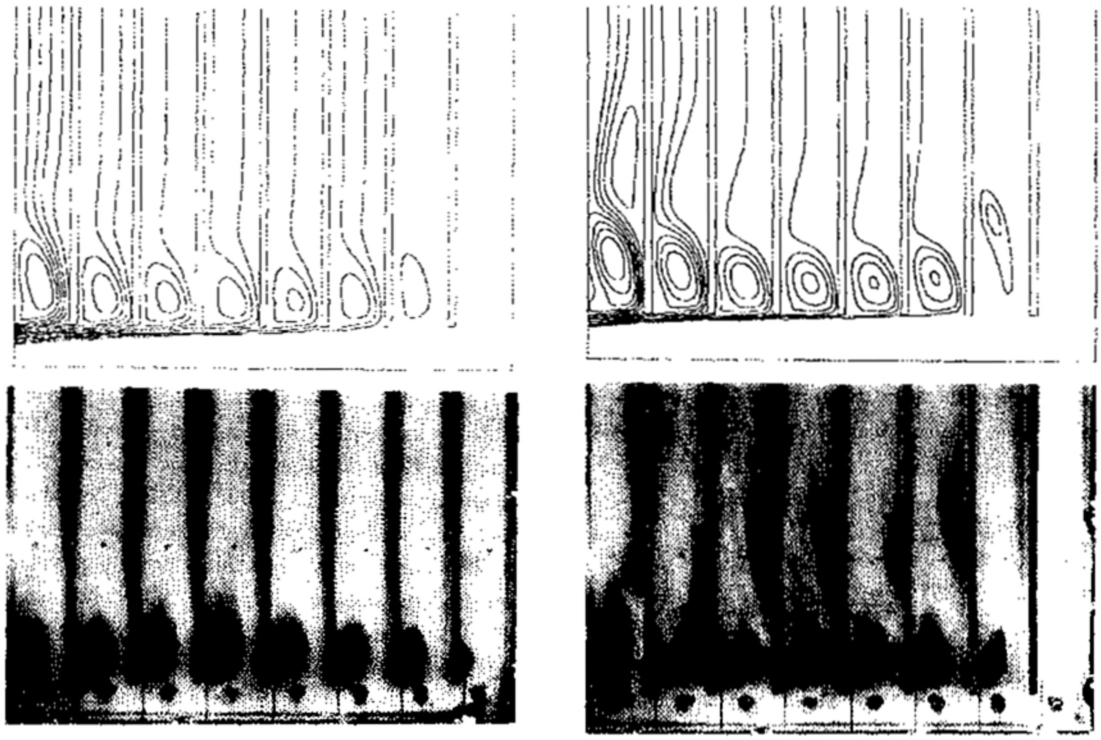


Fig. 9 Comparison of non-dimensionalized center velocity between experiment and numerical results for $Re_w = 450$.



(a) $Re_w = 200$ (b) $Re_w = 560$

Fig. 10 Recirculating flow regime with various Reynolds numbers.

under thermal load. To visualize the recirculating flow, we used a surface film method with the dye coated on the bottom wall of each channel. As water flows, it washed away most of the dye, except in the recirculating flow regime. Figure 10 shows the recirculating flow regime for the numerical and experimetal results at two different Reynolds numbers. For the numerical results in Fig. 10, several values of the streamfunction were selected for a better presentation. A slight difference between the experiments and the computation was observed in the last second channel, which came from the gap between the two dimensional numerical analysis and the flow visualization on the bottom suface. However, both experiments and computations showed a good agreement for the recirculating flow.

For the relatively low Reynolds number flow given in Fig. 10(a), the recirculating flow velocities are very small. The first channel shows a strong recirculating flow even at the neighbor to the wall in the opposite direction of the main flow, which washes away the die near the left wall. For the relatively high Reynolds number flow given in Fig. 10(b), the recirculating flow becomes strong and washes away the dye near the left wall on each channel, which causes off-wall recirculating flow regime. As the Reynolds number increases, the recirculating flow regime becomes longer and another recirculating flow regime is observed in the downstream of the channel. The second recirculating flow regime in the down stream for the low Reynolds number are thin whereas that for the high Reynolds number shows thick layer in both numerical and experimental results. The effect of the Reynolds number on the recirculating flow is dominant in the manifolds. Through the numerical analysis for recirculating flow, α_c and α_d do not affect the recirculating flow as much as the Reynolds number does.

7. Conclusion

In this paper, we addressed the fundamental manifold problem of how to distribute the flow uniformly in the laminar flow conditions. We investigated this problem in three distinct phases, with the following key conclusions:

(a) A new parameter α_d is introduced, which results in the same effect of area ratio for fixed α_d and Reynolds number.

(b) When the Reynolds number and α_c are specified, there is an optimal number of α_d that minimizes the maldistribution of the flow in manifolds. This optimal value of α_d can be predicted by Eq. (6) and is validated by various numerical simulations and flow visualization in the Re_w range of 1-500.

(c) Parabolic profile of flow distribution is a unique characteristic for low Reynolds number flows in manifolds. Also, recirculating flow is strongly dependent on the Reynolds number. These are observed by both numerical simulation and flow visualization.

Acknowledgement

This work was supported by the non-directed research fund from Korea Research Foundation.

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